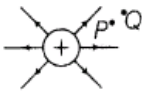


# Electrostatic Potential

## Previous Year Examination Questions

### 1 Mark Questions

1. The figure shows the field lines of a positive charge. Is the work done by the field in moving a small positive charge from Q to P positive or negative? [Foreign 2014]



Ans.

Work done by charge is given by

$W = q$  (potential at Q – potential at P).

where,  $q$  = small positive charge (1/2)

The electric potential at a point distant  $r$  due to the field created by a positive charge  $Q$  is given by

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

$\therefore r_p < r_Q$

$\Rightarrow V_p > V_Q$  (1/2)

So, work done will be negative.

2. For any charge configuration, equipotential surface through a point is a normal to the electric field. Justify. [Delhi 2014]

Ans.

No work is done in moving the test charge from one point of an equipotential surface to the other. (1/2)

$$\therefore W_B - W_A = 0 = -\int \mathbf{E} \cdot d\mathbf{l}$$

$$\Rightarrow \mathbf{E} \cdot d\mathbf{l} = 0$$

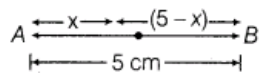
$$\text{Hence, } \mathbf{E} \perp d\mathbf{l} \quad (1/2)$$

3. Two charges  $2 \mu\text{C}$  and  $-2 \mu\text{C}$  are placed at points A and B, 5 cm apart. Depict an equipotential surface of the system. [Delhi 2013C]

Ans.

$$\text{Given, } q_A = 2 \mu\text{C} = 2 \times 10^{-6} \text{ C}$$

$$q_B = -2 \mu\text{C} = -2 \times 10^{-6} \text{ C}$$



$$\text{and } r = 5 \text{ cm}$$

$$\begin{aligned} \therefore \text{Potential, } V &= \frac{2 \times 10^{-6}}{4\pi\epsilon_0 x \times 10^{-2}} \\ &\quad + \frac{-2 \times 10^{-6}}{4\pi\epsilon_0 (5-x) \times 10^{-2}} \\ \therefore \frac{2 \times 10^{-6}}{4\pi\epsilon_0 x \times 10^{-2}} &= \frac{2 \times 10^{-6}}{4\pi\epsilon_0 (5-x) \times 10^{-2}} \quad [\because V = 0] \\ x &= 5 - x \\ x &= 2.5 \quad (1) \end{aligned}$$

4. Why electrostatic potential is constant throughout the volume of the conductor and has the same value as on its surface? [Delhi 2012]

Ans. Since, electric field intensity inside the conductor is zero. So, electrostatic potential is a constant.

$$\begin{aligned} \text{But, } E &= -\frac{\Delta V}{\Delta r} \\ \therefore E &= 0, \Delta V = 0 \\ \text{or } V_2 - V_1 &= 0, V_2 = V_1 \end{aligned}$$

5. Why is the potential inside a hollow spherical charged conductor is constant and has the same value as on its surface? [Foreign 2012]

Ans. Electric field inside the hollow spherical charged conductor is zero. So, no work is done in moving a charge inside the shell.

This implies that potential is a constant and therefore, equal to its value at the surface, i.e.

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{R}$$

6. Why there is no work done in moving a charge from one point to another on an equipotential surface? [Foreign 2012]

Ans.



An equipotential surface is a surface at which every point electric potential is same.

As, work done i.e. moving a charged particle from one point to another is defined as

$$\Delta W = q(\Delta V)$$

On an equipotential surface, the potential remains constant. So,  $\Delta V = \text{zero}$

$$\Rightarrow \text{Work done, } \Delta W = 0 \quad (1)$$

7. A hollow metal sphere of radius 5 cm is charged such that potential on its surface is 10 V. What is the potential at the centre of the sphere? [All India 2011]

Ans.



The electric potential at every point inside the charged spherical shell is same and equal to the electric potential on its surface.

The electric potential at the centre of sphere is 10 V. (1)

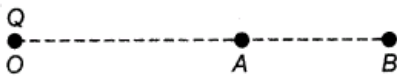
8. Can two equipotential surface intersect each other? Justify your answer. [Delhi 2011 c]

Ans. (i) No, two equipotential surfaces cannot intersect each other because two normals can be drawn at intersecting point on two surfaces which give two directions of E at the same point which is impossible.

(ii) Also, two values of potential at the same point is not possible.

9. A point charge Q is placed at point O as shown in the figure. Is the potential difference ( $V_A - V_B$ )

positive, negative or zero if Q is (i) positive (ii) negative



[All India 2011]

Ans.

Let the distance of points A and B from charge Q be  $r_A$  and  $r_B$ , respectively.

$\therefore$  Potential difference between points A and B

$$V_A - V_B = \frac{q}{4\pi\epsilon_0} \left[ \frac{1}{r_A} - \frac{1}{r_B} \right]$$

As,  $r_A = OA$ ,  $r_B = OB$  and  $r_A < r_B$ ,  $\frac{1}{r_A} > \frac{1}{r_B}$

Therefore,  $\left[ \frac{1}{r_A} - \frac{1}{r_B} \right]$  has positive value.

( $V_A - V_B$ ) depends on the nature of charge q.

(i) ( $V_A - V_B$ ) is positive when  $Q > 0$

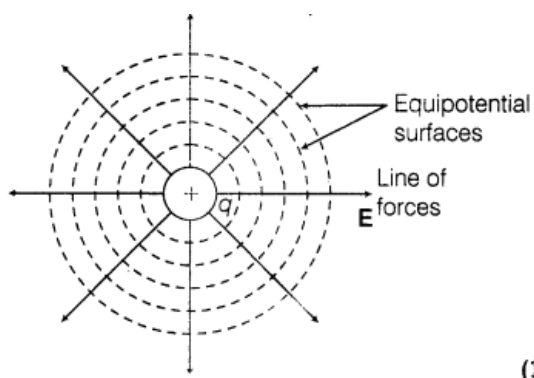
(ii) ( $V_A - V_B$ ) is negative when  $Q < 0$

$$(1/2 + 1/2 = 1)$$

10. Draw equipotential surfaces due to a single point charge. [All India 2011 c]

Ans. Equipotential surfaces due to a single point charge are concentric sphere having charge at the centre.





11. Name the physical quantity whose SI unit is J/C. Is it a scalar or a vector quantity? [All India 2010]

Ans. J/C is the SI unit of **electric** potential. It is a scalar quantity.

12. What is the work done in moving a test charge  $q$  through a distance of 1 cm along the equatorial axis of an electric dipole? [All India 2009]

Ans.

The electric potential at any point on equatorial line due to an electric dipole is equal to zero.

No work is done in moving a test charge along equatorial line, as given below:

$$W = q\Delta V = q(V_B - V_A)$$

$$\therefore V_B = V_A = 0$$

$$W = 0 \quad (1)$$

13. Define the term potential energy for charge  $q$  at a distance  $r$  in an external field. [All India 2009]

Ans. The electric potential energy at any point lying at a distance  $r$  from the source charge  $q$  is equal to the amount of work done in moving unit positive test charge from infinity to that point without any acceleration against of electrostatic force.

14. The potential due to a dipole at any point on its axial line is zero. [All India 2009 C]

Ans. Wrong, as potential due to an electric dipole is zero on equatorial line in spite of axial line. The potential due to a dipole at any point on equatorial line is zero, not an axial line.

15. What is the electric potential due to an electric dipole at an equatorial point? [All India 2009]

Ans. Zero, as potential on equatorial point due to charges of electric dipole, is equal to magnitude but opposite in nature and hence, their resultant is zero

**16.** A  $500 \mu\text{C}$  charge is at the centre of a square of side 10 cm. Find the work done in moving a charge of  $10 \mu\text{C}$  between two diagonally opposite points on the square. [Delhi 2008]

Ans.

💡 Two diagonally opposite points on the square are equidistant from the centre of the square. Therefore, potential at these points due to charge  $q$  will be equal to each other.

Work done,  $W = q\Delta V$   
 $W = q \times 0$   
 $W = 0$

[ $\Delta V = 0$  as the two diagonally opposite points are at the same potential due to  $500 \mu\text{C}$  charge]. (1)

## 2 Marks Questions

17. Two point charges  $q_1$  and  $q_2$  are located at  $r_1$  and  $r_2$ , respectively in an external electric field  $E$ . Obtain the expression for the total work done in assembling this configuration. [Delhi 2014 C]

Ans.

Work done in bringing the charge  $q_1$  from infinity to position  $r_1$

$$W_1 = q_1 V(r_1) \quad \dots (i)$$

(1/2)

Work done in bringing charge  $q_2$  to the position  $r_2$

$$W_2 = q_2 V(r_2) + \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}} \quad \dots (ii)$$

(1/2)

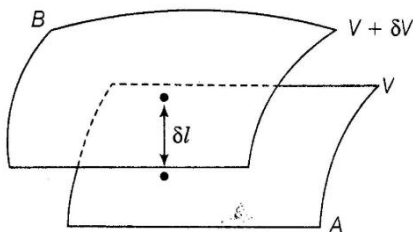
Hence, total work done in assembling the two charges.

$$W = W_1 + W_2 \quad (1)$$

From Eqs. (i) and (ii), we get

$$W = q_1 V(r_1) + q_2 V(r_2) + \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}}$$

18. Two closely spaced equipotential surfaces  $A$  and  $B$  with potentials  $V$  and  $V + \delta V$ , (where  $\delta V$  is the change in  $V$ ) are kept  $\delta l$  distance apart as shown in the figure. Deduce the relation between the electric field and the potential gradient between them. Write the two important conclusions concerning the relation between the electric field and electric potential. [Delhi 2014C]



Ans.

Work done in moving a unit positive charge along distance  $\delta l$ ,

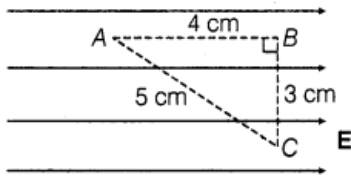
$$|E_l| \delta l = V_A - V_B = V - (V + \delta V) = -\delta V$$

$$E = \frac{-\delta V}{\delta l} \quad (1)$$

19. A test charge  $q$  is moved without acceleration from A to C along the path from A to B and then from B to C in electric field  $E$  as shown in the figure.

(i) Calculate the potential difference between A and C.

(ii) At which point (of the two) is the electric potential more and why?



[Foreign 2009; All India 2012]

Ans.

(i)  $\because$  Electric field intensity and potential difference are related as

$$E = -\frac{\Delta V}{\Delta r}$$

$$\Delta V = -E \Delta r$$

$$\Rightarrow V_C - V_A = -4E \quad (1)$$

$$\left[ \begin{array}{l} \because \text{By pythagoras law, } AC^2 = AB^2 + BC^2 \\ \Rightarrow AB^2 = 5^2 - 3^2 \\ \Delta r = \sqrt{16} \\ \Delta r = 4 \end{array} \right]$$

(ii) As  $V_C - V_A = 4E$  is negative (1)

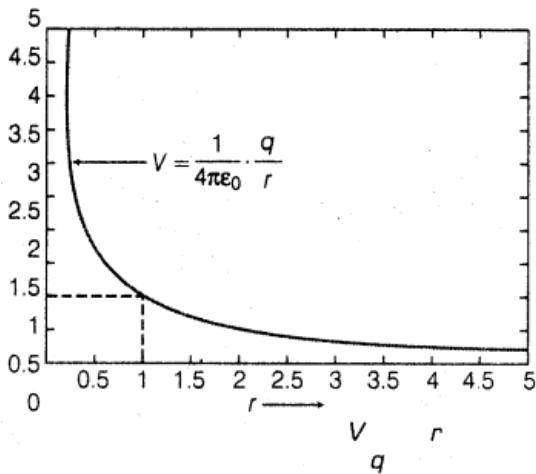
$$\therefore V_C < V_A$$

Potential is greater at point A than point C, as potential decreases along the direction of electric field.

20. Graph the electric potential (V) with distance  $r$  due to a point charge  $Q$ . [Delhi 2012]

Ans.

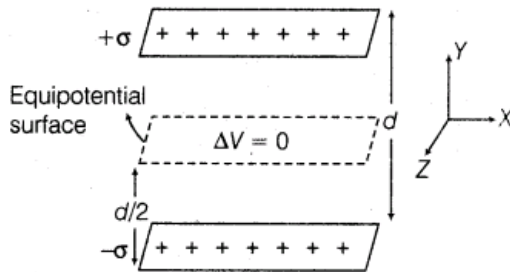
The graph is shown as below:



Potential will be maximum where  $r$  will be minimum (not zero). Zero potential assumed to be at infinity, i.e. at large distance. (1)

21. Two uniformly large parallel thin plates having charge densities  $+\sigma$  and  $-\sigma$  are kept in the  $XZ$  - plane at a distance  $d$  apart. Sketch an equipotential surface due to electric field between the plates. If a particle of mass  $m$  and charge  $-q$  remains stationary between the plates. What is the magnitude and direction of this field? [Delhi 2011]

Ans.



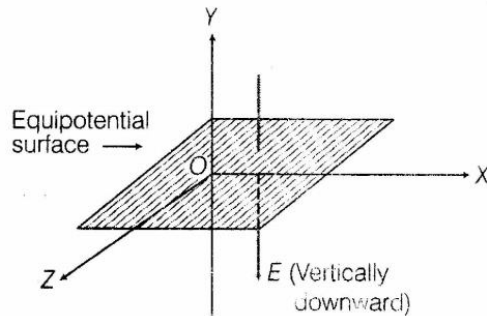
The equipotential surface is at a distance  $d/2$  from either plate in  $XZ$ -plane.  $-q$  charge experiences a force in a direction opposite to the direction of electric field. (1)

$\therefore -q$  charge balances when

$$qE = mg$$

$$E = \frac{mg}{q}$$

The direction of electric field along vertically downward direction. The  $XZ$ -plane is so chosen that the direction of electric field due to two plates is along vertically downward direction, otherwise weight ( $mg$ ) of charge particle could not be balanced. (1)



22. A dipole with its charge  $-q$  and  $+q$  located at the points  $(0, -b, 0)$  and  $(0, +5, 0)$  is present in a uniform electric field  $E$ . The equipotential surfaces of this field are planes parallel to the  $Y$   $Z$ -plane.

(i) What is the direction of the electric field  $E$ ?

(ii) How much torque would the dipole experience in this field? [Delhi 2010 C]

Ans.

The direction of electric field is perpendicular to the equipotential surface.

- (i) The direction of electric field is along X-axis as it should be perpendicular to equipotential surface lying in YZ- plane.

Length of the dipole =  $2b$

As dipole's axis is along the Y-axis.

∴ Electric dipole moment

$$\mathbf{p} = q(2b)\hat{\mathbf{j}} \quad (1)$$

- (ii) Electric field,  $\mathbf{E} = E\hat{\mathbf{i}}$

$$\begin{aligned} \therefore \tau &= \mathbf{p} \times \mathbf{E} \\ &= q(2b)\hat{\mathbf{j}} \times E\hat{\mathbf{i}} \\ &= +2qbE(\hat{\mathbf{j}} \times \hat{\mathbf{i}}) \\ &= 2qbE(-\hat{\mathbf{k}}) \end{aligned}$$

$$\therefore \text{Torque } |\tau| = 2qbE \quad (1)$$

**Alternative method**

$\mathbf{E}$  is directed along X-axis.

Dipole moment  $p = q(2b)$  from  $(0, -b, 0)$  to  $(0, b, 0)$ , i.e. along Y-axis. (1)

∴ Angle between  $\mathbf{p}$  and  $\mathbf{E}$  is  $90^\circ$

$$\begin{aligned} \therefore \text{Torque on dipole} &= \tau_{\max} = pE \sin 90^\circ \\ &= q(2b)E \times 1 \end{aligned}$$

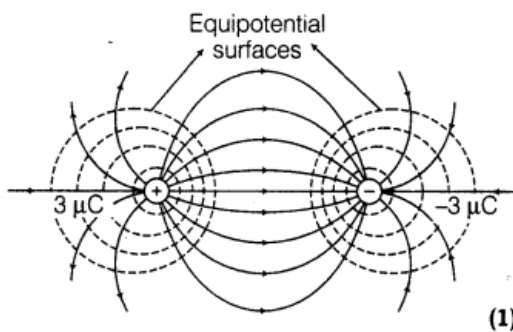
$$\therefore \text{Torque } \tau = 2qbE \quad (1)$$

- 23.** Two point charges  $3 \mu\text{C}$  and  $-3 \mu\text{C}$  are placed at points A and B, 5 cm apart.

- (i) Draw the equipotential surfaces of the system.  
 (ii) Why do equipotential surfaces get close to each other near the point charge? [All India 2011C]

Ans.

- (i) **Equipotential surfaces**



- (ii) Equipotential surfaces get closer to each other near the point charges as strong electric field is produced there.

$$E = -\frac{\Delta V}{\Delta r} \text{ and } E \propto -\frac{1}{\Delta r}$$

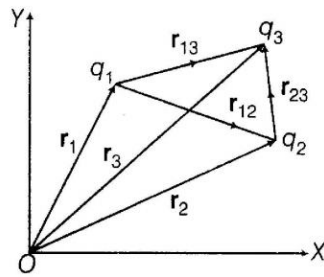
For a given equipotential surface, small  $\Delta r$  represents strong electric field and vice-versa. (1)

24. Find out the expression for the potential energy of a system of three charges  $q_1, q_2$  and  $q_3$  located at  $r_1, r_2$  and  $r_3$  with respect to the common origin O. [Delhi 2010 c]



Ans.

Let three point charges  $q_1, q_2$  and  $q_3$  have position vectors  $\mathbf{r}_1, \mathbf{r}_2$  and  $\mathbf{r}_3$ , respectively.



Potential energy of the charges  $q_1$  and  $q_2$ ,

$$U_{12} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{|\mathbf{r}_{12}|} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{|\mathbf{r}_2 - \mathbf{r}_1|}$$

Similarly,  $U_{23} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_2 q_3}{|\mathbf{r}_3 - \mathbf{r}_2|}$

$$U_{31} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_3}{|\mathbf{r}_3 - \mathbf{r}_1|} \quad (1)$$

$\therefore$  Net potential energy of the system,

$$U = U_{12} + U_{23} + U_{31}$$

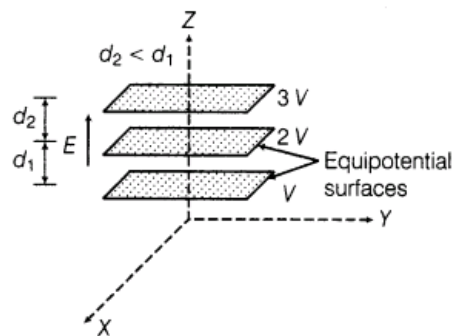
$$U = \frac{1}{4\pi\epsilon_0} \left[ \frac{q_1 q_2}{|\mathbf{r}_2 - \mathbf{r}_1|} + \frac{q_2 q_3}{|\mathbf{r}_3 - \mathbf{r}_2|} + \frac{q_1 q_3}{|\mathbf{r}_3 - \mathbf{r}_1|} \right] \quad (1)$$

25. Draw three equipotential surfaces corresponding to a field that uniformly increases in magnitude but remains constant along Z-direction. How are these surfaces different from that of a constant electric field along Z-direction? [Foreign 2008; All India 2009]

Ans.

Electric field is always directed from higher to lower potential and perpendicular to the equipotential surfaces.

The figure is shown as below :



(1)

In case of constant electric field along Z-direction, the perpendicular distance between equipotential surfaces remains same whereas for field of increasing magnitude, equipotential surfaces get closer as we go away from the origin. In both cases, surfaces be planes parallel to XY- plane.

26.(i) Can two equipotential surfaces intersect each other? Give reasons,

(ii) Two charges  $-q$  and  $+q$  are located at points A  $(0, 0, -a)$  and B  $(0, 0, +a)$  respectively. How much work is done in moving a test charge from point P  $(7, 0, 0)$  to Q  $(-3, 0, 0)$ ? [Delhi 2009]

Ans.(i) No, two equipotential surfaces cannot intersect each other because two normals can be drawn at intersecting point on two surfaces which give two directions of E at the same

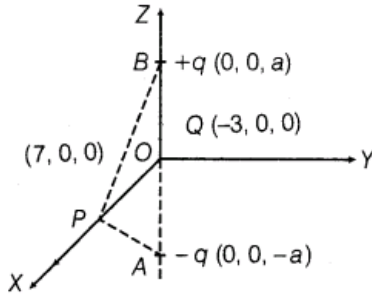
point which is impossible.

(ii)  $\therefore$  Every point on X-axis is on equatorial line of electric dipole (system of two unlike charges).

$\therefore$  Therefore, potential on it is equal to zero.

$\therefore$  No work is done in moving a test charge.

$$W = q \Delta V = q \times 0 = 0. \quad [\because \Delta V = 0]$$



$\therefore$  Work done in moving a charge on an equipotential surface is zero. (1)

27.(i) Can two equipotential surfaces intersect each other? Give reasons, (ii) Two charges + q and - q are located at points A(0, 0, - 2) and B (0, 0, 2) respectively. How much work will be done in moving a test charge from point P(7,0,0) to Q (- 3, 0, 0)? [Delhi 2009]

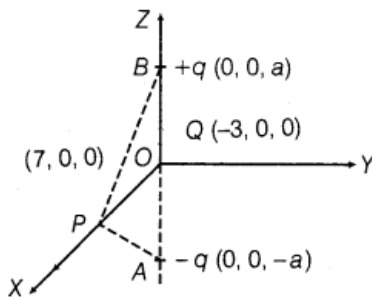
Ans.(i) No, two equipotential surfaces cannot intersect each other because two normals can be drawn at intersecting point on two surfaces which give two directions of E at the same point which is impossible.

(ii)  $\therefore$  Every point on X-axis is on equatorial line of electric dipole (system of two unlike charges).

$\therefore$  Therefore, potential on it is equal to zero.

$\therefore$  No work is done in moving a test charge.

$$W = q \Delta V = q \times 0 = 0. \quad [\because \Delta V = 0]$$



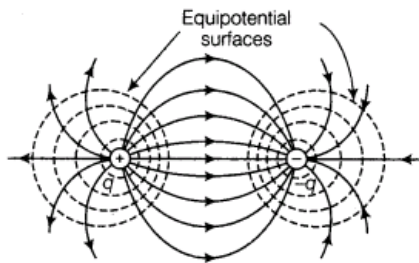
$\therefore$  Work done in moving a charge on an equipotential surface is zero. (1)

28.(i) Write two characteristics of equipotential surfaces.

(ii) Draw the equipotential surfaces due to an electric dipole. [All India 2009 C]

Ans.(i) Two equipotential surfaces do not intersect each other as normals at intersecting points on two surfaces will give two directions of electric field which is impossible.

(ii) Closely spaced equipotential surfaces represent strong electric field and vice-versa.



Equipotential surfaces due to an electric dipole

29. Two point charges,  $q_1 = 10 \times 10^{-8} \text{ C}$  and  $q_2 = -2 \times 10^{-8} \text{ C}$  are separated by a distance of 60 cm in air.

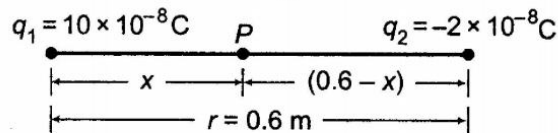
(i) What a distance from the 1st charge  $q_1$  would the electric potential be zero?

(ii) Also, calculate the electrostatic potential energy of the system. [All India 2007, 2008]

Ans.

💡 The electric potential is a scalar quantity. The total potential at a point due to different charges is equal to the sum of individual potentials due to them.

(i) Let electric potential due to two point charges  $q_1$  and  $q_2$  is zero at point  $P$  which lie at distance  $x$  from  $q_1$ .



$$\begin{aligned}
 V_1 + V_2 &= 0 \\
 \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1}{x} + \frac{1}{4\pi\epsilon_0} \cdot \frac{q_2}{(0.6 - x)} &= 0 & (1/2) \\
 \Rightarrow \frac{q_1}{x} + \frac{q_2}{(0.6 - x)} &= 0 \\
 \Rightarrow \frac{10 \times 10^{-8}}{x} + \frac{(-2 \times 10^{-8})}{(0.6 - x)} &= 0 \\
 \Rightarrow \frac{10}{x} = \frac{2}{(0.6 - x)} \\
 6 - 10x &= 2x \Rightarrow 12x = 6 \\
 x &= \frac{1}{2} \text{ m} \\
 x &= 50 \text{ cm} & (1/2)
 \end{aligned}$$

Therefore, electric potential is zero at a distance 50 cm from charge  $q_1$  between two charges.

(ii) Potential energy,  $U = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r}$

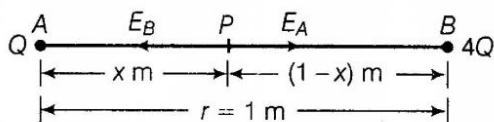
$$= 9 \times 10^9 \times \frac{(10 \times 10^{-8})(-2 \times 10^{-8})}{0.6}$$

$$\begin{aligned}
 U &= -3 \times 10^{-5} \text{ J} \\
 &= -30 \mu\text{J}
 \end{aligned}$$

30. Two point charges  $4Q, Q$  are separated by  $1\text{ m}$  in air. At what point on the line joining the charges, is the electric field intensity zero? Also calculate the electrostatic potential energy of the system of charges taking the value of charge,  $Q = 2 \times 10^{-7}\text{ C}$ . [All India 2008]

Ans.

Let electric field intensity be zero at any point  $P$  lying at a distance  $x\text{ m}$  from charge  $Q$  between the two charges on the line joining them.



Therefore, at point  $P$

$$E_A = E_B$$

$$\frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{x^2} = \frac{1}{4\pi\epsilon_0} \cdot \frac{4Q}{(1-x)^2}$$

$$\Rightarrow \frac{1}{x^2} = \frac{4}{(1-x)^2}$$

$$\left(\frac{1-x}{x}\right)^2 = 4 \quad \text{or} \quad \frac{1-x}{x} = 2$$

$$3x = 1$$

$$x = \frac{1}{3}\text{ m} \quad (1)$$

Electric field intensity  $E$  is zero at a point lying  $x = \frac{1}{3}\text{ m}$  from  $+Q$  charge on the line joining two charges.

Now, electrostatic potential energy

$$U = \frac{1}{4\pi\epsilon_0} \cdot \frac{(4Q)(Q)}{r}$$

$$U = 9 \times 10^9 \times \frac{4 \times (2 \times 10^{-7})^2}{1}$$

$$U = 1.44 \times 10^{-3}\text{ J} = 1.44\text{ mJ} \quad (1)$$

31. Define the dipole moment of an electric dipole. How does the electric potential due to a dipole vary on the dipole axis as a function of  $r$  distance of the point from the mid-point of the dipole at large distances? [All India 2008 C]

Ans. Electric field inside the hollow spherical charged conductor is zero. So, no work is done in moving a charge inside the shell.

This implies that potential is a constant and therefore, equal to its value at the surface, i.e.

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{R}$$

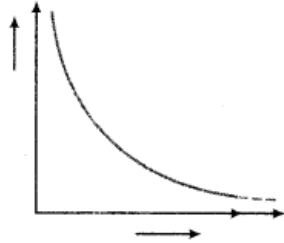
variation of electric potential due to dipole on axial lines when point is at large distance from mid-point of the dipole, occurs as per relation below.

$$V = \frac{1}{4\pi\epsilon_0} \frac{p}{r^2}$$

For given dipole,  $\frac{p}{4\pi\epsilon_0} = \text{constant}$

$$\Rightarrow V \propto \frac{1}{r^2} \quad (1)$$

The variation of electric potential with distance  $r$  is shown as below:



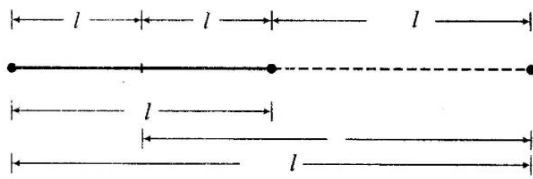
Distance from mid-point of dipole

32. Derive an expression for the electric potential at any point along the axial line of an electric dipole. [Delhi 2008]

Ans.



Consider an electric dipole  $AB$  consists of two point charges  $+q$  and  $-q$  separated by a distance  $2l$ . A point  $P$  be on its axis separated by a distance  $r$  from dipole's centre  $O$ .



Electric dipole moment,

$$p = q(2l) \quad \dots(i)$$

[from  $-q$  charge to  $+q$  charge]

Now, electric potential  $V_A$  at point  $P$  due to  $+q$  charge

$$V_A = \frac{1}{4\pi\epsilon_0} \frac{q}{(r-l)} \quad \dots(ii)$$

Similarly, electric potential  $V_B$  at point  $P$  due to  $-q$  charge

$$V_B = \frac{1}{4\pi\epsilon_0} \frac{-q}{(r+l)} \quad \dots(iii)$$

$\therefore$  Net electric potential at point  $P$  due to two charges

$$\begin{aligned} V &= V_A + V_B = \frac{q}{4\pi\epsilon_0} \left[ \frac{1}{(r-l)} - \frac{1}{(r+l)} \right] \\ V &= \frac{q}{4\pi\epsilon_0} \left[ \frac{(r+l) - (r-l)}{(r-l)(r+l)} \right] \\ V &= \frac{q}{4\pi\epsilon_0} \frac{(2l)}{(r^2 - l^2)} \quad \dots(iv) \end{aligned}$$

[from Eq. (i)  $p = q(2l)$ ]

When  $l \ll r$ ,

$$V = \frac{1}{4\pi\epsilon_0} \frac{p}{r^2} \quad \dots(v)$$

33. Two charges of  $5 \text{ nC}$  and  $-2 \text{ nC}$  are placed at points  $(5 \text{ cm}, 0, 0)$  and  $(23 \text{ cm}, 0, 0)$  in the region of space, where there is no other external field. Calculate the electrostatic potential energy of this charge system. [Delhi 2008 C]

Ans.

Given,  $q_1 = 5 \times 10^{-9} \text{ C}$

$$r_1 = 5 \times 10^{-2} \hat{i} \text{ m}$$

$$q_2 = -2 \times 10^{-9} \text{ C}$$

and  $r_2 = 23 \times 10^{-2} \hat{i} \text{ m}$

$$\begin{aligned} \therefore r_2 - r_1 &= (23 - 5) \times 10^{-2} \hat{i} \\ &= 0.18 \hat{i} \text{ m} \end{aligned}$$

$$\Rightarrow |r_2 - r_1| = 0.18 \text{ m} \quad (1/2)$$

$\therefore$  Electrostatic potential energy, (1/2)

$$\begin{aligned} U &= \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r} = \frac{1}{4\pi\epsilon_0} \times \frac{q_1 q_2}{r} \\ &= 9 \times 10^9 \times \frac{(5 \times 10^{-9})(-2 \times 10^{-9})}{0.18} \end{aligned}$$

$$U = -5.0 \times 10^{-7} \text{ J}$$

$$U = -0.5 \mu\text{J} \quad (1)$$

### 3 Marks Questions

34. A wire AB is carrying a steady current of 12 A and is lying on the table. Another wire CD carrying 5 A is held directly above AB at a height of 1 mm. Find the mass per unit length of the wire CD, so that it remains suspended at its position when left free. Give the direction of the current flowing in CD with respect to that in [Take the value of  $g = 10 \text{ ms}^{-2}$ ] [All India 2013]

Ans.

Force per unit length between the current carrying wires is given as

$$F = \frac{\mu_0}{4\pi} \cdot \frac{2I_1 I_2}{r}$$

where,  $I_1 =$  current in wire,  $AB = 12 \text{ A}$  and  $I_2 =$  current in wire,  $CD = 5 \text{ A}$  and  $r =$  distance between wires  $= 1 \text{ mm} = 1 \times 10^{-3} \text{ m}$

$$\therefore \frac{\mu_0}{4\pi} \frac{2I_1 I_2}{r} = mg$$

Here,  $m$  is mass per unit length.

$$\Rightarrow 10^{-7} \times \frac{2 \times 12 \times 5}{1 \times 10^{-3}} = m \times 10$$

$$\Rightarrow m = 10^{-7} \times \frac{2 \times 12 \times 5}{1 \times 10^{-3}} \times \frac{1}{10}$$

$$= 1.2 \times 10^{-3} \text{ kg/m} \quad (3)$$

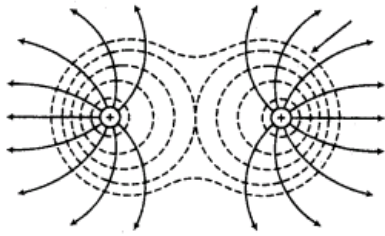
35.(i) Depict the equipotential surfaces for a system of two identical positive point charges placed a distance  $d$  apart.

(ii) Deduce the expression for the potential energy of a system of two point charges  $q_1$  and  $q_2$  brought from infinity to the points with positions and  $r_2$  respectively, in presence of external electric field  $E$ . [Delhi 2010]

Ans.



(i) The figure is shown as below :

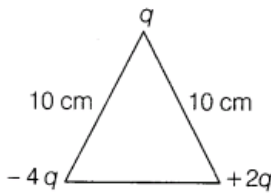


Equipotial surfaces of two identical positive charges

By definition, electric potential energy of any charge  $q$  placed in the region of electric field is equal to the work done in bringing charge  $q$  from infinity to that point and given by  $U = qV$  where,  $V$  is the electric potential (as potential at infinity is assumed to be zero) where, the charge  $q$  is placed. Now, considering the electric potentials at positions  $r_1$  and  $r_2$  as  $V_1$ , and  $V_2$ , respectively. Therefore, total potential energy of the system of two charges  $q_1$  and  $q_2$  placed at points with position vectors  $r_1$  and  $r_2$  in the region of  $E$  is given by  $U =$  work done in bringing charge  $q_1$  from infinite to that position in  $E$  is equal to work done for charge  $q_2$  from infinite to that position in  $E$  + work done to that of charge  $q_2$  at these positions in presence of  $q_1$

$$\begin{aligned} \text{i.e. } U &= q_1 V_1 + q_2 V_2 \\ &= \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{|r_2 - r_1|} \end{aligned}$$

**36.** Calculate the work done to dissociate the system of three charges placed on the vertices of a triangle as shown given in the figure.



Here,  $q = 1.6 \times 10^{-10}$  C. [Delhi 2008]

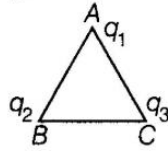
Ans.





The total electrostatic potential energy of this system is

$$U = \frac{1}{4\pi\epsilon_0 R} \left[ \frac{q_1 q_2}{AB} + \frac{q_2 q_3}{BC} + \frac{q_3 q_1}{CA} \right]$$



Work done in making the system  $U =$  Total electrostatic potential energy of the system

Distance,  $r = 10 \text{ cm} = 0.1 \text{ m}$  (1)

$$U = U_{12} + U_{23} + U_{31} \quad (1/2)$$

$$= \frac{1}{4\pi\epsilon_0} \left[ \frac{q(-4q)}{r} + \frac{(-4q)(2q)}{r} + \frac{(2q)(q)}{r} \right]$$

$$= -\frac{1}{4\pi\epsilon_0} \cdot \frac{10q^2}{r}$$

$$= -9 \times 10^9 \times \frac{10 \times (1.6 \times 10^{-10})^2}{0.1}$$

$$U = -1.44 \times 10^{-8} \text{ J} \quad (1/2)$$

$\therefore$  Work done to dissociate the system

$$W = -U$$

$$W = +1.44 \times 10^{-8} \text{ J} \quad (1)$$

37.(i) Two points charges  $q_1$  and  $q_2$  initially at infinity, are brought one by one to points  $P_1$  and  $P_2$  specified by position vectors  $r_1$  and  $r_2$  relative to same origin. What is the potential energy of this charge configuration? Define an equipotential surface.

(ii) Draw schematically the equipotential surface corresponding to a field that uniformly increases in magnitude but remains constant in direction. [Delhi 2008]

Ans.



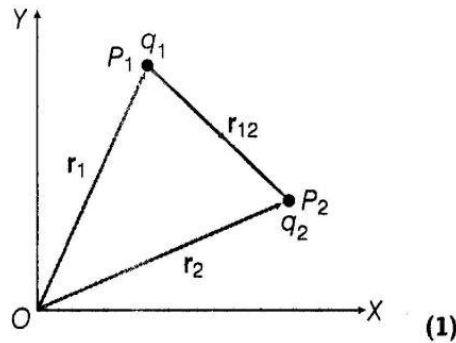


Potential energy of a system of two charges depends upon position vector of one with respect to other.

- (i) Potential energy of the combination of two charges

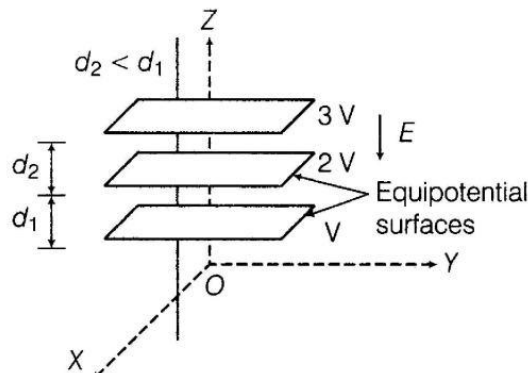
$$U = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{|r_{12}|}$$

$$U = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{|r_2 - r_1|}$$



- (ii) (a) **Equipotential surface** A surface which has equal potential at every point on it, is called equipotential surface. It can be obtained by finding locus or set of points in the region of electric field which are at the same potential. (1)

- (b) The figure of equipotential surface is shown as below :



Equipotential surfaces corresponding to a field that uniformly increases in magnitude (1)

